Abstract—A fuzzy controller has been suggested for attitude control of magnetic actuated satellites in order to calculate the desired mechanical torque based on the attitude error including error in the angles and their rates. The problem of selecting proper magnetic dipole based on the known desired mechanical torque has been investigated, two different methods for the purpose have been suggested and the performance of the attained fuzzy magnetic attitude controllers has been shown under two different simulated conditions.

I. INTRODUCTION

Using magnetic torquers as the sole actuator for attitude control of small satellites despite its benefits has a drawback that is lack of controllability. Mechanical torque can be created only perpendicular to the instantaneous magnetic field vector, thus at each instant the satellite can be controlled only around two axes out of its three perpendicular axes. Fortunately because of the satellite motion, the direction of uncontrollable axis changes by time, therefore the sufficient control torque can be achieved along time.

During last two decades several magnetic attitude controllers have been developed [1]-[5] and some of them have been widely exploited in practical attitude control projects.

The area of magnetic attitude control using intelligent methods is not yet fully explored. Author of [6] has proposed a fuzzy attitude controller for magnetic actuated satellites with two limiting assumptions: bang-off-bang method of actuation, and activating only one torque coil out of the three coils at an instant. These assumptions result in a considerable simplicity in the producible mechanical torque at each instant. He has used an estimation of the producible mechanical torque to calculate the magnetic dipole. In [7] based on a similar limiting assumption, a fuzzy attitude controller has been suggested for spin-stabilized satellites with active magnetic actuation. The authors of [8] have used a neural network to be trained based on a PD controller with constant gains to achieve the three axes stability of micro magnetic actuated satellites. In this work the capability of neural networks to be trained based on a PD controller with the original PD controller. Finally, in [9] a neural network with time varying weights has been utilized as the closed loop near time-optimal controller for rest-to-rest magnetic attitude maneuvers with pre-defined initial and final attitude conditions, hence, it can be used only for a pre-determined attitude maneuver and not for general attitude regulation.

In this work, a fuzzy controller has been suggested to calculate the desired magnetic torque based on the real-time attitude information. This controller is similar to other fuzzy attitude controllers. But the problem arises from the system of actuation, because we cannot create the desired mechanical torque due to the explained problem of uncontrollability. One should calculate the best magnetic dipole to be created in the magnetic coils/rods, which results in the best feasible mechanical torque. For this stage, two approaches has been suggested which in the first one, an analytical approach is used to calculate the feasible mechanical torque and hence the required magnetic dipole; and in the second one, another fuzzy system is responsible to deliver a fairly good magnetic dipole for the mission. Finally the attained controllers have been simulated to depict their performance for two different conditions.

II. SATELLITE STATE EQUATION

The satellite equation of motion is explained using Dynamics and Kinematics equations, where the dynamics equation relates the satellite's angular velocity to the applied mechanical torque, and the kinematics equation relates the instantaneous satellite's attitude to its angular velocity.

For simplicity in the modeling, an inertial pointing satellite has been selected for modeling and simulation of the controller's performance. The satellite dynamics can be represented as [10]

$$\frac{d\omega^B_I}{dt} = -\omega^B_I \times I\omega^B_I + N_{ctrl} + N_{gg} + N_{dist}$$

$$N_{ctrl} = m_{ctrl} \times B$$

where $B, I, \omega^B_I, I, m_{ctrl}, B, N_{ctrl}, N_{gg}$ and $N_{dist}$ refer to body frame, inertial frame, angular velocity of the body frame w.r.t. the inertial frame, satellite inertia tensor, magnetic dipole produced in the magnetic coils, the local geomagnetic field vector, control torque, gravity gradient torque, and disturbance torque respectively. All the vectors are represented in the body frame. In inertial pointing satellites, despite the nadir pointing ones, the gravity gradient torque has an undesired effect of disturbing the satellite with a cyclic nature and is usually categorized as a disturbance torque. One can select a satellite with inertia tensor...
To avoid singularity conditions in modeling the kinematics of the satellite, four Euler parameters known as quaternions denoted by \( q \) are utilized [10].

Describing the rotation of the body frame w.r.t. the inertial frame by a unit vector \( \mathbf{n} \) representing the rotation axis and a scalar \( \epsilon \) representing the rotation angle, the quaternions can be defined as:

\[
\mathbf{q} = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} \cos(\epsilon/2) \\ n_1 \sin(\epsilon/2) \\ n_2 \sin(\epsilon/2) \\ n_3 \sin(\epsilon/2) \end{bmatrix}
\]  

(3)

Where \( \mathbf{n} = [n_1 \ n_2 \ n_3]^T \)

Now, the kinematics equation can be written as [10]:

\[
\begin{align*}
\dot{q}_0 &= -\frac{1}{2}(\omega^{BI})^T[q] \\
\dot{q}_1 &= \frac{1}{2}(\omega^{BI} q_0 - 1) \\
\dot{q}_2 &= \frac{1}{2}(\omega^{BI} q_1 + \omega^{BI} q_3) \\
\dot{q}_3 &= \frac{1}{2}(\omega^{BI} q_2 - \omega^{BI} q_1)
\end{align*}
\]

(5) 

In the last equation, the transpose of \( \omega^{BI} \) is denoted by \( (\omega^{BI})^T \).

In order to initialize the quaternions based on available initial Euler angles \( \phi, \theta, \psi \), the following set of equations is utilized.

\[
\begin{align*}
q_0 &= \cos(\frac{\psi}{2}) \cos(\frac{\theta}{2}) \cos(\frac{\phi}{2}) + \sin(\frac{\psi}{2}) \sin(\frac{\theta}{2}) \sin(\frac{\phi}{2}) \\
q_1 &= \cos(\frac{\psi}{2}) \cos(\frac{\theta}{2}) \sin(\frac{\phi}{2}) - \sin(\frac{\psi}{2}) \sin(\frac{\theta}{2}) \cos(\frac{\phi}{2}) \\
q_2 &= \cos(\frac{\psi}{2}) \sin(\frac{\theta}{2}) \cos(\frac{\phi}{2}) + \sin(\frac{\psi}{2}) \cos(\frac{\theta}{2}) \sin(\frac{\phi}{2}) \\
q_3 &= \sin(\frac{\psi}{2}) \cos(\frac{\theta}{2}) \cos(\frac{\phi}{2}) - \cos(\frac{\psi}{2}) \sin(\frac{\theta}{2}) \sin(\frac{\phi}{2})
\end{align*}
\]

(7)

And similarly the Euler angles are related to the quaternions through the following equations.

\[
\begin{align*}
\tan(\phi) &= \frac{2(q_2 q_3 - q_1 q_0)}{q_0^2 - q_1^2 - q_2^2 + q_3^2} \\
\sin(\theta) &= -\frac{2(q_2 q_3 + q_1 q_0)}{q_0^2 - q_1^2 - q_2^2 + q_3^2} \\
\tan(\psi) &= \frac{2(q_1 q_3 + q_2 q_0)}{q_0^2 + q_1^2 - q_2^2 - q_3^2}
\end{align*}
\]

(8)

Finally choosing the four elements of the quaternions and the three elements of the angular velocity vector as the states and the three elements of the magnetic dipole as the controls and discretizing the resulting time-varying nonlinear system for \( N \) time steps with equal sampling time of \( h \) will result in the discrete form of the system state equation.

\[
x[k+1] = f(x[k], u[k], k, h), k = 0,1, ..., N-1
\]

(9)

With

\[
x[0] = x_0
\]

(10)

Where \( f, x \) and \( u \) are:

\[
\begin{align*}
&f = h \begin{bmatrix}
-\omega_x q_1 - \omega_y q_2 - \omega_z q_3 \\
\omega_x q_0 + \omega_y q_3 - \omega_z q_2 \\
\omega_y q_0 - \omega_x q_3 + \omega_z q_1 \\
\omega_z q_0 + \omega_x q_2 - \omega_y q_1
\end{bmatrix} + \begin{bmatrix}
0 \\
q_1 \\
q_2 \\
q_3
\end{bmatrix} \\
x = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \\
u = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}
\end{align*}
\]

(11) 

III. FUZZY CONTROLLER

As mentioned earlier, in this work the fuzzy control of magnetic actuated satellites has been divided into two stages; in the first stage one should calculate the desired mechanical torque based on the attitude information, i.e. the states, and in the second stage the control magnetic dipole should be calculated to create producible mechanical torque.

A. First Stage: Calculating Desired Mechanical Torque

Denoting the desired mechanical torque by \( \mathbf{N}_{des} \), three elements of this vector can be calculated through three multi-input single-output (MISO) fuzzy controllers which get the angle’s error and the rate’s error and generate required torque around the corresponding axis to regulate these errors.

In this work a Sugeno type fuzzy inference method is selected and the block diagram of the controller is shown in fig. 1. The membership functions of the inputs are depicted in fig. 2 and fig. 3 and the rules output values are \(-2,-1,0,1,2\). The output of the controller will get any value in the interval of \([-2, 2]\). The rules of the inference engine are listed in table I.
TABLE I
RULES OF THE FIRST STAGE FUZZY CONTROLLER

<table>
<thead>
<tr>
<th>Attitude Error</th>
<th>Rate Error</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>2</td>
<td>-2</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

For example, if the rate error is large-negative (i.e. -2) and the attitude error is small-negative (i.e. -1) then the desired torque value should be large-positive (2).

B. Second Stage: Calculating Magnetic Dipole

Approach 1

Having calculated the desired mechanical torque, the problem is calculating a magnetic dipole to produce the most similar but achievable mechanical torque. In this approach an analytical method inspired by [5] will be used to derive the achievable torque and the required magnetic dipole.

The policy is projecting the desired mechanical torque vector into the plane perpendicular to the local geomagnetic vector. The projected vector will be the achievable mechanical torque $\mathbf{N}_{achv}$.

$$\mathbf{N}_{achv} = (\mathbf{E} - \mathbf{u}_B \mathbf{u}_B^T) \mathbf{N}_{des}$$  \hspace{1cm} (14)

Where $\mathbf{E}$ and $\mathbf{u}_B$ are identity matrix and unit vector in the direction of geomagnetic vector, respectively. Now, having the direction of geomagnetic field vector and the achievable mechanical torque one can calculate the direction of required control dipole using the following equation.

$$\mathbf{u}_m = \mathbf{u}_B \times \mathbf{N}_{achv}$$  \hspace{1cm} (15)

The unit vector of $\mathbf{N}_{achv}$ is shown by $\mathbf{u}_{N_{achv}}$. Finally, selecting a constant gain $K$, the applicable magnetic control is

$$\mathbf{m}_{ctrl} = K \mathbf{u}_m$$  \hspace{1cm} (16)

C. Second Stage: Calculating Magnetic Dipole

Approach 2

In the second approach, another fuzzy system will be utilized to calculate the magnetic dipole based on the available desired mechanical torque and the present local geomagnetic field vector.

The relation between the mechanical torque, the geomagnetic field vector, and the magnetic dipole vector is described by the following equation.

$$\begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix} = \begin{bmatrix} m_y B_z - m_z B_y \\ m_z B_x - m_x B_z \\ m_x B_y - m_y B_x \end{bmatrix}$$  \hspace{1cm} (17)

Let's reshape the equation in a slightly different form

$$\begin{bmatrix} N_x' \\ N_y' \\ N_z' \end{bmatrix} = \begin{bmatrix} m_y' B_z - m_z' B_y \\ m_z' B_x - m_x' B_z \\ m_x' B_y - m_y' B_x \end{bmatrix}$$  \hspace{1cm} (18)

Actually $\mathbf{m}_i' = \mathbf{m}_i$ for $i = x, y, z$ but in this approach let's consider them separate. One can derive some fuzzy rules based on equation 18 to derive the control dipole, e.g. if a large negative $N_x$ is desired and $B_y$ is small positive then $m_y'_{-}$ should be large negative to fulfill the demand. The rules required to calculate $m_i'$ are mentioned in table II. The same set of rules can be used to calculate $m_i''$ providing feeding $-B_i$ instead of $B_i$ for $i = x, y, z$.

The block diagram of the process is depicted in fig. 4. Using the six MISO fuzzy systems, one can derive each of these six $m_i'$ and $m_i''$. Finally, using the averaging equation of 19, the applicable control dipole can be calculated.

$$m_i = \frac{m_i' + m_i''}{2}, \quad i = x, y, z$$  \hspace{1cm} (19)

Fig. 4. Block diagram of the second stage fuzzy controller
The inputs are shown in Fig. 5 and 6. Utilized [11]. Characteristics of the orbit and the satellite have been performed for the following two different conditions:

**Conditions A:** Initial Euler angles = [−70, 80, 90] deg.
Initial angular velocity = [1, 0.5, −1] × 10^{-3} rad/s
Approach 1 constant gain = 1000
Approach 2 constant gain = 0.1
Constant disturbance torque = [0, 0, 0] N.m

**Conditions B:** Initial Euler angles = [90, 30, 80] deg.
Initial angular velocity = [-0.5, -1, -0.5] × 10^{-3} rad/s
Approach 1 constant gain = 1000
Approach 2 constant gain = 0.1
Constant disturbance torque = [1, 1, 1] × 10^{-3} N.m

The results are depicted in Fig. 7 to 10. The performances of both of the approaches are satisfactory. The steady state error of the simulation of conditions A is less than 0.1 deg and under the disturbed simulation of conditions B it is less than 8 deg. Both of the controllers have been able to force the attitude to converge to the desired direction after about 1.5 orbital periods.

### TABLE III
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>orbit semi-major axis</td>
<td>7000 km</td>
</tr>
<tr>
<td>Inclination</td>
<td>96°</td>
</tr>
<tr>
<td>right ascension of the ascending node</td>
<td>0°</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0</td>
</tr>
<tr>
<td>orbital period</td>
<td>5828 s</td>
</tr>
<tr>
<td>saturation limit of each magnetic coil</td>
<td>10 A.m²</td>
</tr>
<tr>
<td>satellite inertia tensor</td>
<td>diag(1,1,1) kg.m²</td>
</tr>
</tbody>
</table>

Using a typical fuzzy controller, the desired mechanical torque has been calculated, but the problem is selecting a magnetic dipole to be applied and generate the best achievable torque. Two different methods for calculating the applicable magnetic dipole has been proposed and the performance of both of the approaches has been shown under two simulations with different conditions. Interestingly their performances were comparable to the conventional magnetic attitude controllers while the simplicity and flexibility of the controllers due to the fuzzy base are their considerable benefit.

### IV. CONCLUSION

The inference method used in the second phase fuzzy systems is Sugeno type and the output of each rule is a member of set {−2, −1, 0, 1, 2}. The membership functions of the inputs are shown in Fig. 5 and 6.

Briefly, using approach 2 of the suggested fuzzy attitude controller, one should calculate the desired magnetic torque from the block diagram shown in Fig. 1 (introduced as the first stage fuzzy controller) based on the available attitude errors, then feed the desired torque along with the local geomagnetic field vector to the block diagram of Fig. 4 (the second stage fuzzy controller) and calculate the control dipole. Of course a constant gain should be used to scale the output of the block diagram of Fig. 4 and adjust the value of the control.

**D. Simulation**

To simulate the proposed fuzzy attitude controllers, the geomagnetic field model IGRF2000 with degree 13 has been utilized [11]. Characteristics of the orbit and the satellite under the simulation are listed in Table III. The simulation has been performed for the following two different conditions:

**Conditions A:** Initial Euler angles = [−70, 80, 90] deg.
Initial angular velocity = [1, 0.5, −1] × 10^{-3} rad/s
Approach 1 constant gain = 1000
Approach 2 constant gain = 0.1

### REFERENCES


