Transfer of correlations in neural oscillators
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Abstract: How do the nonlinear dynamics of neurons combine with the statistics of their common forcing to determine correlations in their response? We examine how phase oscillators transfer correlations in their output spikes. Simulations and general linear response theory indicate that efficiency of correlation transfer is largely independent of firing rate, but depends strongly on time scale of measurement and internal dynamics of the neuron. Type I (integrating) neurons have higher correlation efficiency than Type II (resonating) neurons at long times, contrasting findings at short time scales (Marella and Ermentrout '08).

3) Background: Setup

\[ \omega + \sigma_\mu \sqrt{1 - c} \cdot \xi(t) \]
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Inputs
- phase speed
- noise variance
- fraction of noise in common

phase oscillators

6) At long time $S(\omega, \alpha)$ maximized for Type I (theta model), minimized for Type II

Unlike $v$ and $CV$, $dV/d\alpha$ changes robustly with $\alpha$

Response of the neuron to increased input $\mu$

\[ d\theta = d\omega \mu z(t) dt \]

A perturbation argument in small $\mu$

\[ \frac{d\theta}{d\mu} = \mu \int_0^\infty Z(\theta) p_\theta \frac{d\theta}{d\mu} \]

Phase resetting curve (PRC):

\[ Z(\theta) = -\alpha \sin(\theta) + (1 - \alpha)(1 - \cos(\theta)), \quad 0 \leq \alpha \leq 1 \]

$\alpha = 0$ (theta model) $\alpha = 0.5$ $\alpha = 1.0$ (Type II)

4) Linear response theory yields expression for susceptibility

(c.f. Lindner et al 2005)

Spike train and Fourier transform

\[ \tilde{y}(\omega) = \sum \mathcal{F}(t) e^{-i \omega t} \]

Linear ansatz in Fourier domain

\[ \tilde{y}(\omega) = \theta(\omega) + \sqrt{\alpha} \tilde{w}(\omega) \]

Cross-spectrum:

\[ \mathcal{C}_w(k) = \tilde{y}(\omega) \tilde{y}(f) = A_{\omega, \omega} \tilde{w}(\tilde{w})(\tilde{w}) \tilde{w}(f) \]

In the limit $T \rightarrow \infty$

\[ \int C_w(\omega) d\omega = C_{\omega}(0) = c \sigma \int \frac{dV}{d\omega} \]

5) Simulations confirm $\rho^2 \propto \frac{1}{\sigma}$ varies linearly with $c$

8) No firing rate dependence at large $T$: contrasts with LIF model

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