Wave-driven vortex dynamics in the near-shore region

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Physical processes in near-shore region

- Within 1 km of coastline
- Typical depth – 1-10m
- Shoaling and breaking of surface waves
- Turbulent (quadratic) drag
The near-shore current system

- Obliquely breaking waves create alongshore current
- Alongshore current transports sediment, can be exploited to prevent erosion
- Displaced/varying alongshore current associated with rip currents
Overview

- Current dislocation on barred beaches is still inadequately explained
- Main idea: Alongshore variation of wave energy on scale of wave groups can produce current dislocation
  - Idealized experiments
  - Near-shore current system is non-turbulent
Alongshore current

- Obliquely breaking waves create alongshore current.
- The momentum transferred shoreward by surface waves is defined as “radiation stress”.
- Convergence of radiation stress transfers momentum to the mean current.
- Longuet-Higgins (1970) used the momentum balance between radiation stress convergence and bottom friction to solve for current.

Result: current is strongest at locations of strongest wave breaking

Bühler and Jacobson, 2001
Comparison with experiments (Duck, NC)

- Ok on linear beach (e.g. Santa Barbara)
- Not so good on barred beach…
- Other authors incorporate horizontal mixing (Longuet-Higgins 1970), enhanced friction due to turbulence (Church and Thornton 1993), wave rollers (Ruessink et al 1998, many others)

Church and Thornton (1993)
New mechanism for current dislocation

- Directional/frequency spreading can produce alongshore inhomogeneous wave breaking on O (100 m) (Reniers et al. 2002, 2004)

- Inhomogeneous wave breaking produces vortex dipoles, which locate current in trough

- Numerical model, idealized studies
Breaking wave packets produce vortex dipoles

Peregrine (1998)
Behavior of vortices on a sloping beach

- Model vortex as axisymmetric vortex ring
- Properties of vortex ring determined by local slope of beach

\[ U = \frac{\Gamma}{4\pi} \left( \frac{\nabla h_S}{h_S} \times \mathbf{\hat{z}} \right) \ln \left( \frac{8}{b_0 h_{S_0}^{1/2} \left| \nabla h_S \right|^{3/2}} \left( \frac{h_S^{3/2}}{\nabla h_S} \right) \right) \]
Planar vs. barred beach
Rigid-lid approximation

For low Froude number flow (\( U << \sqrt{gh} \)) we have

\[
\nabla \cdot (h_s \mathbf{u}) = 0
\]

\[
\left( \frac{\partial}{\partial t} + (\mathbf{u}^L \cdot \nabla) \right) \left( \frac{\nabla \times \mathbf{u}}{h_s} \right) = \frac{1}{h_s} \nabla \times \left( \mathbf{B} - \frac{1}{h_s} \nabla \cdot \mathbf{S} \right)
\]

The last term is the “radiation stress” of Longuet-Higgins (defined on next slide)

We now describe the flow by the single dynamic equation

\[
\frac{Dq}{Dt} = -\frac{1}{h} \nabla \times \left( c_f \frac{\mathbf{u} | \mathbf{u} |}{h} + \frac{1}{h} \nabla \cdot \mathbf{S} \right)
\]

\[
q \equiv \nabla \times \mathbf{u} \quad \frac{1}{h}
\]
Rotational part of radiation stress (BJ01)

\[
S_{ij} \equiv h_S u_i' u_j' + \delta_{ij} \frac{g}{2} h'^2
\]

\[
- \frac{1}{h_S} \nabla \cdot S = \frac{\partial p}{\partial t} - F - \frac{1}{2} \nabla |u'|^2
\]

\[
F \equiv \frac{k}{h_S} \nabla \cdot \left( h_S \frac{k}{k^2} E \right)
\]

In the presence of steady waves, only F makes a contribution to the curl of the momentum on the previous slide. F is non-zero in the absence of dissipation.
Wave parameterization

- Geometric ray theory
- Waves “break” when they exceed saturation threshold

\[
\frac{d\mathbf{x}}{dt} = \Omega_k \\
\frac{d\mathbf{k}}{dt} = -\Omega_x \\
\Omega(\mathbf{k}, x) = \sqrt{gh_s} \kappa \\
\frac{\partial A}{\partial t} + \nabla \cdot (c_g A) = 0
\]

Waves are forced to break when they exceed a saturation threshold: following LH70,

\[
A = \min(A^{sat}, A^{unsat}) \\
A^{sat} = A(\alpha h_s) \\
\alpha = 0.41
\]
Numerical model: governing equations

\[ \frac{Dq}{Dt} = -\frac{1}{h} \nabla \times F - \frac{c_f}{h} \nabla \times \frac{\mathbf{u} \mid \mathbf{u}}{h} \]

\[ \nabla \cdot \left( \frac{\nabla \psi}{h} \right) = hq \]

\[ \mathbf{u} = \frac{1}{h} \nabla \perp \psi \]

\[ 0 \leq x \leq D, 0 \leq y \leq L \]

\[ \psi(x,0) = \psi(x,L) \]

\[ \psi(0,y) = 0 \]

\[ \psi(D,y) = M \psi(D,y) \]

M is the “Dirichlet-to-Neumann” map (DtN) of the operator

\[ \nabla \cdot \left( \frac{\nabla \psi}{h} \right) \]

for some specified \( h(x) \) and boundary conditions at infinity.
Idealized experiments on current dislocation

- Linear vs. barred topography
- Homogeneous vs. inhomogeneous (packet)
Vortex dipole

Packet of waves produces vortex dipole
Idealized study: barred beach, homogeneous waves

- No current dislocation
Idealized study: barred beach, homogeneous waves

No current dislocation
Idealized study: barred beach, homogeneous waves

No current dislocation
Idealized study: barred beach, homogeneous waves

No current dislocation
Idealized study: barred beach, homogeneous waves

No current dislocation
Idealized study: linear beach, packet of waves

- Modest current dislocation
Idealized study: linear beach, packet of waves

- Modest current dislocation
Idealized study: linear beach, packet of waves

Modest current dislocation
Idealized study: linear beach, packet of waves

Modest current dislocation
Idealized study: linear beach, packet of waves

- Modest current dislocation
Idealized study: barred beach, packet of waves

Marked current dislocation
Idealized study: barred beach, packet of waves

Marked current dislocation
Idealized study: barred beach, packet of waves

Marked current dislocation
Idealized study: barred beach, packet of waves

Marked current dislocation
Idealized study: barred beach, packet of waves

Marked current dislocation

![Graph showing mean alongshore velocity vs distance from shoreline]
Idealized study: barred beach, packet of waves

Marked current dislocation

![Graph showing mean alongshore velocity (m/s) vs distance from shoreline (m) over 1 hour.](image-url)
Longer time velocity observations, $c_f = 0.014$
Near-shore current system is non-turbulent

- Shallow water = 2-D fluid with varying fluid depth
- Can we have upward energy cascade with physical parameters typical of the beach (Peregrine 1998, 1999)?
2D Turbulence - phenomenology

- Conservation properties imply “inverse” cascade of energy, “direct” cascade of enstrophy.
- Both cascades “arrested” by dissipative processes.
- At which length scale do these dissipative processes act? Do they depend on the strength of forcing?

Vallis, 2006
Near-shore current system is non-turbulent

Grainik et al. study cascade phenomenology of

\[
\frac{\partial \xi}{\partial t} + u \cdot \nabla \xi = F_\xi + D_\xi
\]

\[
D_\xi = \nabla \times (-C_d |u| u)
\]

\(\xi\) is forced at large wavenumber \(k_f\). The upward energy cascade will be arrested at a scale \(k_a\) which is \textit{independent} of the strength of forcing.
Near-shore current system is non-turbulent.
Near-shore current system is non-turbulent

Grainik et al. estimate

\[ k_a \approx 51C_d \]

\[ k_a \approx 51 \frac{c_f}{h} \]

If \( c_f = 0.01 \), then

\[ k_a h = 0.5 \]

But we only model mean motions for which

\[ k_a h < 1 \]

(on surf zone, mean flow also subject to littoral friction)
Conclusions and on-going work

- New numerical model for study of near-shore region, with open boundary and parameterized forcing
- Vortex dipole is shown to provide a mechanism for current dislocation
- Surf zone is non-turbulent (2D)
- Long-term goal: When will low-frequency wave energy produce vortex dipoles capable of dislocating current?
Idealized experiment: sinusoidal forcing

- directional spreading and/or frequency spreading cause “groupiness”
- Alongshore and time variation on the order of 100 m
Vortex dipole: sinusoidal forcing
Current maximum vs. $f_b$