Impact of Single-Neuron Dynamics on Transfer of Correlations from Common Input

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Common input from population coding creates correlations

These cells receive common synaptic input from this cell in a previous layer

Information flow
Common input from population coding creates correlations

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\[ \Rightarrow P(x_1, x_2 | s) \neq P(x_1 | s)P(x_2 | s) \]
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Information flow

shared fraction \( \sim c \)

\[ n_1 = 3 \]

\[ n_2 = 4 \]
Common input from population coding creates correlations

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How does this relationship depend on single cell dynamics?

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Linear response theory: for small noise, linear in \( c \) (cf. Lindner et al. 2005)

\[
\rho_T \equiv \frac{\text{Cov}(n_1,n_2)}{\sqrt{\text{Var}(n_1)} \sqrt{\text{Var}(n_2)}} \approx c S_T
\]
How does this relationship depend on single cell dynamics?

One way to characterize neurons: resting-to-spiking excitability

\[
C \frac{dV}{dt} = I - g_K n^4 (V - E_K) - g_{Na} m^3 h (V - E_{Na}) - g_L (V - E_L) + I_{app}
\]

\[
\frac{dn}{dt} = \left( n_\infty (V) - n \right) / \tau_n (V)
\]

\[
\frac{dm}{dt} = \left( m_\infty (V) - m \right) / \tau_m (V)
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\[
\frac{dh}{dt} = \left( h_\infty (V) - h \right) / \tau_h (V)
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Figures: Guillaume Lajoie, UW
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Type I excitability

Type II excitability

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How does this relationship depend on single cell dynamics?

Compare Type I vs. Type II dynamics:

• Stimulus selectivity and neurocomputational properties: integrator vs. resonator (Agüera y Arcas et al. 2003; Mato and Samengo 2008)
• Synchronization properties (Ermentrout 1996; Hansel et al. 1995; Wang and Buzsáki 1996)
• Type I/II transition can be effected by regulating slow potassium currents (Ermentrout et al. 2001), such as by neuromodulators (Steifel et al. 2008a, 2008b) or level of background activity (Prescott et al. 2008)

Figures: Guillaume Lajoie, UW
A neural model that shows both Type I and Type II excitability

Connor-Stevens model
(Connor and Stevens 1971, Connor et al. 1977, Rush and Rinzel 1994)

$$C \frac{dV}{dt} = I - g_K n^4 (V - E_K)$$
$$- g_{Na} m^3 h (V - E_{Na}) - g_L (V - E_L)$$

**K⁺**
$$\frac{dn}{dt} = (n_\infty(V) - n) / \tau_n(V)$$

**Na⁺**
$$\frac{dm}{dt} = (m_\infty(V) - m) / \tau_m(V)$$
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Hodgkin-Huxley
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\]

\[
K^+ \quad \frac{dn}{dt} = \left( n_\infty (V) - n \right) / \tau_n (V)
\]

\[
Na^+ \quad \frac{dm}{dt} = \left( m_\infty (V) - m \right) / \tau_m (V)
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Na^+ \quad \frac{dh}{dt} = \left( h_\infty (V) - h \right) / \tau_h (V)
\]

\[
K^+_{tr} \quad \frac{da}{dt} = \left( a_\infty (V) - a \right) / \tau_a (V)
\]

\[
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Hodgkin-Huxley + transient A-current
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Correlation transfer efficiency reverses, based on time scale

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Barreiro et al., J Neurophysiology 2012
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Type II cells transfer more correlation at short time scales; Type I cells transfer more correlation at long time scales.

Barreiro et al., J Neurophysiology 2012
How can we understand this mathematically? Reduced model for ODEs near a limit cycle

Type I excitability (SNIC)

Type II excitability (Hopf)
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If the neuron is firing periodically ("tonic" firing)

\[ \frac{d\theta}{dt} = \omega + Z(\theta)I_{fluc}(t) \]

\( \theta = 2\pi \rightarrow \text{spike} \)

\( Z(\theta) = \text{phase response curve (PRC)} \)

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Type I excitability (SNIC)

\[Z(\theta) = 1 - \cos \theta\]

Type II excitability

\[Z(\theta) = -\sin \theta\]

Type I excitability (SNIC)

Type II excitability

Rinzel and Ermentrout '89
Ermentrout and Kopell '84
Ermentrout '96
Not just an academic exercise: PRCs can be measured (and modulated) in real neurons

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Prescott et al, J Neurophys, 2008

Stiefel et al., PLoS One, 2008
Analytical calculations in phase models

\[ T \gg 1 \rightarrow \rho_T \approx c \frac{\langle Z \rangle^2}{\langle Z^2 \rangle} = cS \]

but

\[ T \ll 1, \rho \approx cT \left(1 - \frac{\langle Z \rangle^2}{\langle Z^2 \rangle}\right) = cT (1 - S) \]

Marella and Ermentrout, PRE, 2008
Barreiro et al., PRE, 2010
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Type I cells are more efficient at transferring correlated input at long time scales, Type II at short time scales.

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In sinusoidal phase oscillators:

\[ \rho_T \]

\[ \log_2 T \]

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In sinusoidal phase oscillators:

Ermentrout et al., PRE, 2008
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Barreiro et al., J Neurophys 2012
Another prediction tool: Use common input spike-triggered average (STA)

\[ \text{Cov}_T(n_1, n_2) = T \int_{-T}^{T} C_{12}(t) \left(1 - \frac{t}{T}\right) dt \]

\[ C_{12}(t) \propto c\left(K \ast \tilde{K}\right)(t); \quad K(t) = \text{STA}(t), \tilde{K} = \text{STA}(-t) \]

\[ \text{STA}(t) = \frac{1}{N_{sp}} \sum_{k=1}^{N_{sp}} I_c(t_k - t) \]
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\]

- Can be used in the subthreshold (excitable) regime
- Time window \((T)\)-specific prediction

Bair et al. 2001, Gabbiani and Koch 1998, Ostojic 2009; (but see Hong et al. 2012)
STA is very predictive of correlation transfer

\[ \text{Cov}_T (n_1, n_2) = T \int_{-T}^{T} C_{12} (t) \left(1 - \frac{t}{T}\right) dt \]

\[ C_{12} (t) \propto c (K * \tilde{K})(t); \]

\[ K (t) = STA(t), \tilde{K} = STA(-t) \]

superthreshold (mean-driven)

subthreshold (fluctuation-driven)
Correlation transfer modulates downstream firing rate

\[ \tau \dot{V} = -V + I(t) \]

Type II cells will be more effective than Type I cells at driving short time constant neurons.

Type I cells will be more effective than Type II cells at driving long time constant neurons.
Modulatory effect seen across upstream operating regimes

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mean-driven upstream layer

fluctuation-driven upstream layer
Modulatory effect seen across upstream operating regimes

$$\tau \dot{V} = -V + I(t)$$

*Type I/Type II switch occurs at biophysically meaningful timescale:* $\tau \sim 5 - 10$ ms

Destexhe et al. 2003; Prescott and De Koninck 2009

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**mean-driven upstream layer**

**fluctuation-driven upstream layer**
**On-going work:** a novel excitability mechanism: “Type IV”

\[
\dot{V} = \frac{1}{\epsilon_V} \left( V - V^3/3 + 2/3 - n^2 - z + I_{\text{app}} \right)
\]

\[
\dot{n} = \epsilon_n \left( k_m (V - V_0) \left( 1 + \frac{a}{1 + e^{(4k/a)(V_0 - V + 0.2)}} \right) + n_0 - n \right)
\]

\[
\dot{z} = \epsilon_{Ca} \left( \frac{k}{1 + e^{-5(V - e_{Ca})}} - 3 - z \right); \quad \epsilon_{Ca} \ll \epsilon_n
\]

Similar to models in:

---

**Type I/II**
\[ (n_0 = 0.8) \]

**Type IV**
\[ (n_0 = -0.5) \]
On-going work: a novel excitability mechanism: “Type IV”

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\dot{V} = \frac{1}{\varepsilon_V} \left( V - V^3/3 + 2/3 - n^2 - z + I_{\text{app}} \right) \\
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\dot{z} = \varepsilon_C a \left( \frac{k}{1 + e^{-(V-a_C)}} - 3 - z \right); \quad \varepsilon_C a \ll \varepsilon_n
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Similar to models in:

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![Type I/II](image1.png)

Type I/II: \(Ca = -2.2, I_{\text{app}} = -2\)

\(n_0 = 0.8\)

![Type IV](image2.png)

Type IV: \(Ca = -3, I_{\text{app}} = -2.5\)

\(n_0 = -0.5\)
Effect of lower nullcline branch on spike shape: after-depolarization potentials (ADP)

Type I/II

Type IV
In some cases, the STA effectively predicts covariance

\[
Cov_T(n_1, n_2) = T \int_{-T}^{T} C_{12}(t) \left(1 - \frac{t}{T}\right) dt
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C_{12}(t) \propto c(K * \tilde{K})(t);
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\[
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What about $\rho_T$?

$T \gg 1 \rightarrow \rho_T \approx c \frac{\langle Z \rangle^2}{\langle Z^2 \rangle} = cS$

$T \ll 1, \rho \approx cT \left(1 - \frac{\langle Z \rangle^2}{\langle Z^2 \rangle}\right) = cT(1 - S)$

Very low correlation transfer (5%, 2.5% respectively)
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Very low correlation transfer (5%, 2.5% respectively)

(compare to CS: 40-80%)
**Hypothesis:** for systems with strong time scale separation, limited-support PRC $\Rightarrow$ low $\rho_T$
**Hypothesis:** for systems with strong time scale separation, limited-support PRC => low $\rho_T$

$$Z(x) = \begin{cases} \hat{Z}\left(\frac{x}{\epsilon}\right), & 0 \leq x < \epsilon \\ 0, & \epsilon < x < 1 \end{cases}$$

$$\rho_Z = c \frac{\langle Z \rangle^2}{\langle Z^2 \rangle} = c \epsilon \frac{\langle \hat{Z} \rangle^2}{\langle \hat{Z}^2 \rangle} = \epsilon \rho_{\hat{Z}}$$

(compare w/ CS:)
Is the PRC prediction accurate?

\[ T \gg 1 \rightarrow \rho_T \approx c \frac{\langle Z \rangle^2}{\langle Z^2 \rangle} = cS \]

\[ T \ll 1, \rho \approx cT \left(1 - \frac{\langle Z \rangle^2}{\langle Z^2 \rangle}\right) = cT(1 - S) \]

\[ S_{I/II} \approx 0.002 \]

\[ S_{IV} \approx 0.04 \]

\[ \rho_{I/II} > \rho_{IV} \text{ but } S_{I/II} < S_{IV} \]
Possible issues:
• relaxation oscillator with strong time scale separation: insensitive to noisy current except at specific times in cycle
• very hard to get long $T$ statistics
• very hard to get joint statistics
• How long is long enough (for $T$)?
In Conclusion

- We study common input correlation transfer in both conductance-based (Connor-Stevens) and phase-oscillator models, focusing on the transition from Type I to Type II neural dynamics.

- Type II neurons are more correlated at short time scales, but Type I neurons are more correlated at long time scales.

- The Type I/Type II transition can modulate downstream firing rate at biophysically relevant timescales.

- Common input spike-triggered average methodology generalizes well to other excitability types (Type IV).

- PRC-based predictions less accurate: more study needed in relaxation oscillators.

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